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LATERAL BOUNDARIES AND THE HEAT-INDUCED FLOW IN THE TROPICAL ATMOSPHERE

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ABSTRACT

The effect on the heat-induced circulation of lateral vertical boundaries is studied by means of a simple model. The basic state in the absence of boundaries is a Hadley circulation driven by a narrow band of heating at a specified latitude. Lateral friction is included in the model. This has little effect on the meridional circulation but smooths out the zonal flow. The eastern boundary modifies the Hadley flow through planetary waves needed to make the flow at the boundary meridional, and the flow pattern is similar to that of the east Pacific trades. The western boundary generates a Kelvin wave in order to make the total flux normal to the boundary vanish. This can give rise to patterns similar to ones encountered in the Indian Ocean.

1. INTRODUCTION

There has been a recent upsurge of interest in the heat-induced circulation of the tropical atmosphere, particularly as simple models (Matsuno 1966, Webster 1972, Gill 1980) give a good description of the flow patterns that are encountered. However, the majority of such studies have been for flow in a domain of infinite horizontal extent or far a periodic domain. This is reasonable for the upper level flow, but it is clear from observed low level wind patterns that mountain ranges and escarpments such as the Andes and the eastern margin of Africa strongly affect the low-level flow. In this paper, the effect on the flow of vertical meridional boundaries will be studied as a model of such effects. Only one aspect of the boundary effects appears to have been studied in the past, namely the intense low-level southerly jet found along the east coast of Africa (Findlater, 1977, Hart et al., 1978). The boundaries also have large scale effects, which take the form of reflected long Kelvin and planetary waves needed to satisfy the condition of no mass flow through the boundary. The modelling of these effects is well established in the context of the equatorial ocean circulation (Gill, 1975, McCreary, 1981), and allows the large scale effects of boundaries to be described very simply.

During the northern summer the low-level winds over the western Indian ocean consist in the mean of south-easterlies in the southern hemisphere and south-westerlies in the northern hemisphere. One effect of the mountains of East Africa appears to be to link these two flow regions by means of a strong southerly boundary jet. This flow has been described using simple models by Anderson (1976) and by Bannon (1979, 1982). It has also been modelled numerically by Krishnamurti and Wong (1979) and by Krishnamurti et al. (1983). The

dynamics of the boundary jet were studied by Hart (1977).

Anderson and Bannon were concerned with describing the coastal jet resulting from the interaction of a zonally symmetric Hadley circulation with a western boundary. In their models both the heating and the cooling are externally specified forcing terms, usually with the heating concentrated north of the equator, and the cooling south of the equator. The external specification of the cooling, with a given geographical distribution, resulted in a number of unrealistic features in their solutions: (i) a cold core troposphere in the cooling region, together with (ii) high surface pressures relative to mid-latitudes, and therefore (iii) westerly trade winds on the southern flank of the cooling region.

These unrealistic features of the Hadley circulation in the models of Anderson and Bannon can be traced back to the external specification of the cooling. They are not found in the present model, where the cooling is through a self-adjusting Newtonian cooling of the large scale flow. The external specification of the cooling results in a cold troposphere, and is thus incompatible with the idea of enhanced radiation from a hot troposphere. The reason why the heating should be externally specified but the heating described in terms of properties of the large scale flow is that the heating is due to a small-scale process which is not explicitly described, namely convection in cumlonimbus clouds, whereas the cooling is a large scale process resulting from the properties of the large scale flow.

Including the cooling as well as the heating in the forcing term gives the models of Anderson and Bannon another peculiar property which is important from the point of view of boundary effects. Since the vertical velocity is proportional to the net heating or cooling at

any point, models where both heating and cooling are specified in the forcing term Q must satisfy $\iint Q dx dy = 0$ in order that the vertical motions average to zero over the globe. In order to satisfy this requirement Anderson and Bannon worked mainly with forcing functions Q which were antisymmetric about the equator. It is obvious that antisymmetric heating functions generate zonal velocities and pressures antisymmetric about the equator. The linear forced Hadley circulations in the models of Anderson and Bannon therefore satisfy $\int u dy = 0$. It will be shown that this is a special case in which the large scale effects of the western boundary considered here are zero.

The effects of an eastern boundary were taken into account by Gill (1985), but not described explicitely. The main effect is the reflection of the Kelvin wave in a series of planetary waves. This causes the trade winds and the pressure contours to curve as one approaches the boundary from the west, and the winds are purely meridional at the boundary, in good qualitative agreement with observations.

The model is derived in the next section. It is based on the model of Gill (1980), but generalised to allow for the presence of lateral diffusion. The effects of lateral boundaries will be demonstrated for an otherwise x-independent flow, the Hadley circulation obtained in section 3.

2. The Model

The model is basically that of Gill (1980), subsequently used and extended by Heckley and Gill (1984), Gill and Phlips (1986), and by others, and will therefore only be described briefly. The model describes the response of a resting atmosphere to deep heating. Mean winds and low level heating are not considered. The flow is assumed

to be weak, and the governing equations are linearized. The model is simple because the solutions are separable in z, with a modal structure in the vertical. Moreover, the flow in the troposphere is well represented by a single vertical mode (Silva Dias and Bonatti, 1985) because of the simple vertical structure of the heating, which is strongest in the middle troposphere and becomes weak near the surface and near the tropopause (Houze, 1982; Hantel and Baader, 1978).

With the vertical structure of the flow determined from observation, the problem is to find its horizontal structure. The details of the vertical structure only affect the equations for the horizontal structure through a single constant, the wave speed. This may be taken advantage of to simplify the mathematics in the model without any loss of generality. Thus it will be assumed here that the fluid is incompressible and uniformly stratified (N^2=constant), with the surface at z=0 and a rigid lid at the tropopause $z=\pi$. The heating, vertical velocity, and potential temperature then have the same vertical structure, with a maximum in the middle troposphere and zeros at the surface and at the tropopause. The pressure and horizontal velocities also have identical vertical structures, with maxima at the surface and at the tropopause, and a change of sign in the middle troposphere,

$$Q, w, \Theta = (Q', w', \Theta') \sin z \qquad 0 \le z \le \pi \qquad (1)$$

$$\phi, u, v = (\phi', u', v') \cos z$$

In previous theoretical models the damping processes have usually been modelled using simple linear laws: Rayleigh friction $-\epsilon \underline{u}$ in the momentum equations and Newtonian cooling $-\epsilon\Theta$ in the thermodynamic

equation. The damping processes are poorly understood, but whether they result from large-scale eddies, mixing by cumulus clouds, boundary layer turbulence, or some combination of these processes, it seems clear that they will cause lateral mixing of momentum and temperature. The simplest, and classical way to model lateral mixing by eddies is by linear terms

$$A\nabla^2\underline{u} \quad , \quad A\nabla^2\theta \tag{2}$$

in the momentum and thermodynamic equations, where A is an eddy diffusion coefficient which is much larger than the molecular diffusion coefficient. Here we are concerned with problems where the zonal scale is much larger than the meridional scale, so that we may assume that $v \ll u$, and that $\partial^2/\partial x^2 \ll \partial^2/\partial y^2$ (Gill and Phlips, 1986). The important contributions to the lateral mixing (2) therefore come from the terms

$$A\partial^2 u/\partial y^2 \quad , \quad A\partial^2\theta/\partial y^2 \tag{3}$$

in the zonal momentum and thermodynamic equations respectively.

If we non-dimensionalise the governing equations as in Gill (1980), assume the flow to be in hydrostatic balance, linearise the momentum and thermodynamic , equations, make the long wave approximation, and introduce the vertical structure (1), we find

$$\begin{aligned} -\tfrac{1}{2}yv &= -\partial\phi/\partial x - \epsilon u + A\partial^2 u/\partial y^2 \\ \tfrac{1}{2}yu &= -\partial\phi/\partial y \\ w &= Q - \epsilon\Theta + A\partial^2\theta/\partial y^2 \\ \partial u/\partial x + \partial v/\partial y + w &= 0 \\ \Theta &= -\phi \end{aligned} \tag{4}$$

after dropping the primes. The variables x and y have been non-dimensionalised by the equatorial Rossby radius a, $a = (C/2\beta)^{1/2}$

where C is the speed of long gravity waves. With $C \sim 50$ ms^{-1} and 2.3×10^{-11} (ms)$^{-1}$ the equatorial Rossby radius is $a \sim 1000$ km or about 10°. We now eliminate w and Θ from equations (4), and rewrite them in terms of new dependent variables q and r,

$$q = \phi + u \quad , \quad r = \phi - u \tag{5}$$

giving

$$\begin{aligned} -A\partial^2 q/\partial y^2 + \epsilon q + \partial q/\partial x + \left(\frac{\partial}{\partial y} - \frac{1}{2}y\right) v &= -Q \\ -A\partial^2 r/\partial y^2 + \epsilon r - \partial r/\partial x + \left(\frac{\partial}{\partial y} + \frac{1}{2}y\right) v &= -Q \\ \left(\frac{\partial}{\partial y} + \frac{1}{2}y\right) q + \left(\frac{\partial}{\partial y} - \frac{1}{2}y\right) r &= 0 \end{aligned} \tag{6}$$

We see in equations (6) the index raising and index lowering operations for the parabolic cylinder functions $D_n(y)$ (Abramowitz and Stegun, 1965). These functions satisfy

$$\frac{dD_n}{dy} + \frac{1}{2}y D_n = n D_{n-1} \quad , \quad \frac{dD_n}{dy} - \frac{1}{2}y D_n = -D_{n+1} \tag{7}$$

and the first few functions are

$$D_0, D_1, D_2, D_3 = \left(1, y, y^2-1, y^3-3y\right) \exp\left(-\tfrac{1}{4}y^2\right) \tag{8}$$

In the absence of lateral diffusion, equations (6) have modal solutions

$$\begin{aligned} q &= q_{n+1}(x)\, D_{n+1}(y) \\ v &= v_n(x)\, D_n(y) \\ r &= r_{n-1}(x)\, D_{n-1}(y) \end{aligned} \quad , \quad n = -1, 0, 1, 2, \ldots \tag{9}$$

where it is understood that quantities with a negative index are zero. As it stands, the problem (6) cannot be solved in terms of the modes (9) because, unlike sines and cosines, the second derivative of a

parabolic cylinder function is not proportional to the function itself, but satisfies

$$\frac{d^2 D_n}{dy^2} - \frac{1}{4}y^2 D_n = -\left(n+\tfrac{1}{2}\right) D_n \tag{10}$$

This difficulty can be bypassed when A is much smaller than ϵ by assuming that, instead of being constant, the Rayleigh friction and Newtonian cooling increase slightly with latitude,

$$\epsilon = \epsilon^*\left[1+\left(\frac{A}{\epsilon^*}\right)\frac{1}{4}y^2\right] \quad , \quad A/\epsilon^* \ll 1 \tag{11}$$

The equations (6) then become

$$\begin{aligned} -A\left(\frac{\partial^2}{\partial y^2}-\frac{1}{4}y^2\right)q + \epsilon^* q + \frac{\partial q}{\partial x} + \left(\frac{\partial}{\partial y}-\frac{1}{2}y\right)v &= -Q \\ -A\left(\frac{\partial^2}{\partial y^2}-\frac{1}{4}y^2\right)r + \epsilon^* r - \frac{\partial r}{\partial x} + \left(\frac{\partial}{\partial y}+\frac{1}{2}y\right)v &= -Q \\ \left(\frac{\partial}{\partial y}+\frac{1}{2}y\right)q + \left(\frac{\partial}{\partial y}-\frac{1}{2}y\right)r &= 0 \end{aligned} \tag{12}$$

If the heating Q is expanded in terms of parabolic cylinder functions,

$$Q = F(x)\sum_{n=0}^{\infty} Q_n D_n(y) \tag{13}$$

then equations (12) have solutions in terms of the normal modes (9). Using (7), (9), (10), and (13) in (12) gives

(i) Kelvin mode (n=-1)

$$\epsilon^*\left(1+A/2\epsilon^*\right)q_0 + \partial q_0/\partial x = -Q_0 F(x) \quad , \quad r = v = 0 \tag{14}$$

(ii) Mixed mode (n=0)

$$v_0 = Q_1 F(x) \quad , \quad q = r = 0 \tag{15}$$

(iii) Planetary modes ($n \geq 1$)

$$\begin{aligned} \epsilon^*\left[1+n\left(\frac{2n+2}{2n+1}\right)\frac{A}{\epsilon^*}\right]q_{n+1} + c_n\frac{\partial q_{n+1}}{\partial x} + \left(c_n\frac{A}{2}q_{n+1}\right) &= c_n\left(Q_{n-1}+nQ_{n+1}\right)F(x) \\ v_n = \left[Q_{n-1}+(n+1)Q_{n+1}\right]F(x) + 2(n+1)\epsilon^*\left[1+(n+\tfrac{1}{2})\frac{A}{\epsilon^*}\right]q_{n+1} \\ r_{n-1} = (n+1)\,q_{n+1} \end{aligned} \tag{16}$$

where $c = -(2n+1)^{-1}$ is the speed of long planetary waves, and the bracketed third term in the first of (16) is a very small damping term that will eventually be neglected. It is clear from (14) and (16) that, since A/ϵ^* is small, the low n wave modes are little affected by the lateral eddy diffusion. Only the higher wave modes feel the lateral diffusion, which acts to increase the wave damping.

3. THE HADLEY CIRCULATION

The response to a zonally symmetric heating distribution, or Hadley circulation, provides a useful starting point to study the effects of lateral boundaries. Moreover, the Hadley circulation solution is readily comparable with models by Dickinson (1971), Anderson (1976), and others, and with observations.

The heating distribution is taken to be

$$Q = \alpha D_0 \left[\alpha (y - y_0)\right] \tag{17}$$

where y_0 is the latitude of maximum heating, and the width of the heating region is proportional to α^{-1}. With this heating distribution the total amount of heating is independent of α and y_0. The heating distribution (17) can be put in the form (13), with F(x)=1, and

$$Q_{n+1} = \frac{1}{n+1}\left[\frac{\alpha^2}{1+\alpha^2} y_0 Q_n + \frac{1-\alpha^2}{1+\alpha^2} Q_{n-1}\right]$$
$$Q_0 = \left(\frac{2\alpha^2}{1+\alpha^2}\right)^{1/2} \exp\left(\frac{-1}{4}\frac{\alpha^2}{1+\alpha^2} y_0^2\right) \tag{18}$$

as shown in Phlips and Gill (1986). For zonally symmetric heating the zonal derivatives in equations (14)-(16) become negligible, so that the equations reduce to algebraic equations with the solutions

(i) Kelvin mode

$$\epsilon^* q_0 = -Q_0 / (1 + A/2\epsilon^*) \tag{19}$$

(ii) Mixed mode

$$v_0 = Q_1 \quad , q_1 = 0 \tag{20}$$

(iii) Planetary modes ($n \geq 1$)

$$\epsilon^* q_{n+1} = c_n (Q_{n-1} + n Q_{n+1}) \Big/ \left[1 + n\left(\frac{2n+2}{2n+1}\right)\frac{A}{\epsilon^*}\right]$$

$$v_n = c_n \left[Q_{n-1} - (n+1) Q_{n+1}\right] + \left(\frac{2n+2}{2n+1}\right)\left(\frac{A}{\epsilon^*}\right)\epsilon^* q_{n+1} \tag{21}$$

$$n_{n-1} = (n+1)\, q_{n+1} \quad , \quad c_n = -(2n+1)^{-1}$$

The flow associated with heating centred on the equator, and one Rossby radius north of the equator, is shown in figure 1. In both cases the heating region is fairly narrow (α=3) so that a large number of modes (about 30) is required to obtain a smooth representation of the fields. The non-dimensional lateral diffusion coefficient is A=0.01, which is small compared to ϵ^* (ϵ^*=0.1) as required for our method to be valid. This corresponds to a dimensional diffusion coefficient A $O(10^5)\,m^2/s$, which is the order of magnitude found by Hart (1977) to be necessary to give the Somali jet the right width in a frictional model.

The top diagrams in figure 1 show the steamfunction, which is obtained by multiplying the meridional velocity by sin z. The circulation features low-level flows towards the heating region from both north and south, a narrow heating region with large upward velocities, and broad regions on either side with much smaller downward motions. When the heating is centred off the equator, the Hadley cell on the equatorial side is much stronger than that on the poleward side, in agreement with observations. The profiles of vertical velocity, meridional velocity, and pressure relative to the equatorial though are also in good qualitative agreement with the

observations summarised by Riehl (1979).

A comparison with Hadley circulation solutions without lateral diffusion (not shown, see Phlips (1986)) shows that the zonal velocity is the only field to be significantly affected by the lateral diffusion. The pressure, vertical and meridional velocities are nearly identical to those found in the absence of lateral diffusion. When the heating is centred off the equator, the low-level westerlies found between the equator and the heating region are much reduced by the lateral diffusion. But the 'trade winds' poleward and across the equator from the heating region are virtually unchanged, which provides some reassurance that the poleward increase of the Rayleigh friction and Newtonian cooling has no substantial side effects.

4. THE EFFECTS OF AN EASTERN BOUNDARY

In this section the effects of the Andes on the low-level heat induced flow over the Pacific will be modelled by introducing an eastern meridional boundary at $x = X_E$ in an otherwise zonally symmetric Hadley circulation. Linear models of the Hadley circulation have been obtained and described in section 3 and all that remains to be done is to satisfy the boundary condition introduced by the eastern boundary. At the boundary the zonal velocity must vanish,

$$u = 0 \quad \text{at} \quad x = X_E \quad \Rightarrow \quad q = n \quad \text{at} \quad x = X_E \tag{22}$$

The forced Hadley circulation consists of Kelvin, mixed, and long planetary modes. However, the only zero frequency, westward decaying modes are the long planetary modes. The boundary condition (22) must therefore be satisfied by superimposing some combination of the decaying long planetary wave modes

$$q_{n+1} = a_n \exp\left[-\epsilon(x-X_E)/c_n\right] \qquad (23)$$

$$r_n = \Sigma(n+1)\epsilon\, q_{n+1} \quad , \quad r_{n-1} = (n+1)\, q_{n+1} \quad , \quad n = 1,2,3,\ldots$$

(for the case without lateral diffusion) on the forced solution.

For the forced planetary modes the boundary condition (22) is easily satisfied by adding free solutions (23) with amplitude equal to minus the amplitude of the forced solution. The forced mixed mode flow already satisfies $q=\pi=0$, and is therefore unaffected by the presence of the eastern boundary.

When an eastward propagating Kelvin mode reaches the eastern boundary it is reflected in a series of planetary modes. The only non-zero property of the Kelvin mode is q_0, and this determines the amplitude of the gravest planetary mode since $\pi_0 = q_0$ in order to satisfy (22). But the long planetary modes satisfy $=(n+1)q$, $n=1,2,3,\ldots$, which provides the following recursion relation for the amplitudes of all the long planetary modes excited by the incident Kelvin wave,

$$q_{n+1} = q_{n-1}/(n+1) \qquad \text{at } x = X_E \qquad (24)$$

The low-level flow is shown in figure 2 for heating centred on the equator and one Rossby radius north of the equator, and an eastern boundary at x=0. The effects of the boundary are confined to a region near the boundary whose zonal extent is inversely proportional to the magnitude of the friction and cooling. In the examples shown this distance is of the order of five Rossby radii, or about 5000 km. As one approaches the boundary from the west, the easterly trade winds become more and more meridional, until at the boundary the winds are purely meridional and towards the heating region. Away from the equator the winds are nearly geostrophic, and the pressure contours

reflect the curvature of the winds. This curvature of the trade winds and pressure contours is one of the characteristic features of the observed mean surface circulation, as shown in Riehl (1979). The simple model seems to provide a good first order description of the low-level flow over the eastern Pacific.

The westward curvature of the flow near the boundary is due to the reflection of the forced Kelvin wave. This can be seen in figure 3 where the forced Kelvin wave and its reflection are shown separately from the forced planetary waves and their reflections. The equatorward flow near the boundary is seen to feed the westward Kelvin mode flow.

With the inclusion of lateral friction, the normal condition to apply at the meridional boundary is one of no slip. If this condition is applied, the effect is only noticeable in a thin boundary layer adjacent to the wall. This boundary layer is called a Munk boundary layer (Munk 1950), or is a hybrid between a Munk layer and a Stommel layer, and is described at length by Pedlosky (1979). Anderson (1976) has shown that similar boundary layers are found on an equatorial beta-plane. In the boundary layer, the meridional component of the flow is geostrophic and the flow is non-divergent. The vorticity equation takes the approximate form

$$A v_{xxx} - \epsilon v_x - \frac{1}{2} v = 0 \qquad (25)$$

For values typically used of $\epsilon = 0.1$ and $A \sim 1/200$ (corresponding to dimensional A of $10^5 m^2 s^{-1}$), the boundary layer width is around 200 km. (25) is easily solved in terms of exponentials, one solution of the three applying to the eastern boundary layer. This solution serves to reduce the meridional velocity to zero over the 200 km wide layer, but otherwise has little effect. There seems little point in considering

details of this correction in the present context.

5. THE EFFECTS OF A WESTERN BOUNDARY

The effects of the mountains of East Africa on the low-level heat-induced flow over the Indian ocean will be modelled in this section by having a western boundary running north-south at $x = X_w$ in an otherwise zonally symmetric Hadley circulation. The forced flow impinging on the western boundary will excite decaying eastward propagating wave modes. In the low frequency limit these consist of the Kelvin mode and short planetary modes, that decay within a short distance of the boundary. In the steady state limit, the short waves become a boundary jet represented by the two eastward-decaying solutions of (25).

The low-level flow near the African coast does have the properties of a boundary jet, and is described in detail by Findlater (1977) and by Hart et al (1978). The role of the boundary jet is to close the circulation, by connecting the regions of easterlies in the interior to the regions of westerlies. The models of Anderson (1976) and Bannon (1979, 1982) have shown that the boundary jet is the result of the eastward propagating short planetary waves, and such linear models show that the effects of the boundary are confined to the jet, except for the single condition discussed below.

Here we shall adopt the opposite approach, and model the large scale effects of the western boundary without considering the details of the boundary jet. The long wave equations are ideally suited for this purpose as they filter out the short waves making up the boundary layer jet and simplify the mathematics. In the long wave approximation the only effect of the western boundary is to cause a

reflected, decaying Kelvin wave to appear from the boundary

$$q = q_{or}\, D_0(y) \exp\left[-\epsilon(x - X_w)\right] \quad , \quad n = v = 0 \quad , \quad x > X_w \tag{26}$$

Mathematically the problem reduces to finding the amplitude of the reflected Kelvin wave, q_{or}, given the magnitudes of the forced Kelvin wave, q_{of}, and the incident planetary waves. To do this one has to take into account the physical function of the boundary layer, even though it is not explicitly described. The boundary layer jet acts to transfer mass between latitudes. The condition determining the large-scale flow is thus the fact that there can be no flow through the boundary. If only the large-scale, long waves are being considered this condition must be taken in integral form

$$\int_{-\infty}^{\infty} \mu \, dy = 0 \tag{27}$$

at the western boundary $x = X_w$. In terms of q and n this gives

$$q_{or} = -q_{of} - \sum_{n=1}^{\infty} \left(q_{n+1} \int_{-\infty}^{\infty} D_{n+1}\, dy - n_{n-1} \int_{-\infty}^{\infty} D_{n-1}\, dy \right) \Big/ \int_{-\infty}^{\infty} D_0(y)\, dy \tag{28}$$

but for the planetary waves $n_{n-1} = (n+1) q_{n+1}$, and the integrals satisfy

$$\begin{aligned} \int_{-\infty}^{\infty} D_0(y)\, dy &= \sqrt{2\pi} \\ \int_{-\infty}^{\infty} D_{n+1}(y)\, dy &= n \int_{-\infty}^{\infty} D_{n-1}(y)\, dy \end{aligned} \tag{29}$$

so that (27) becomes

$$q_{or} = -q_{of} + \sum_{n=1}^{\infty} q_{n+1} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} D_{n-1}(y)\, dy \tag{30}$$

and since the integrals of D_i are zero for i odd,

$$q_{on} = -q_{of} + \left(q_2 + q_4 + 3.q_6 + 3.5.q_8 + 3.5.7.q_{10} + \ldots \right) \qquad (31)$$

and the q's on the right hand side are obtained from the forced circulation.

The low-level flow is shown in figure 4 for a narrow heating region (α=3) centred on the equator, and centred one Rossby radius north of the equator, the Hadley circulation obtained in section 3. The western boundary is located at x= -15, x and y being scaled in Rossby radii. The large scale effects of a western boundary are less spectacular than those of an eastern boundary, but they extend over a much longer zonal distance. This is due to the higher speed of Kelvin waves. The reflected Kelvin wave strengthens the westerly flow between the equator and the heating region, and weakens the easterly trade winds to the north and south.

Maps of monthly mean low-level streamlines by Ramage (1971) and Riehl (1979), do indeed show westerly winds in the equatorial western Indian ocean, but comparisons with the present, highly idealised model are not very easy. Near the coast the present model does not compare well with the observations because the boundary jet has not been modelled. Moreover, non-linear effects may well be important in the coastal jet (Anderson, 1976), and so a linear steady-state model of the coastal boundary layer may not give a very good description of the flow.

The mean sea level pressure for July is shown in Van de Boogaard (1977). Near the equator the pressure in the Indian ocean drops as one moves eastward from the coast. In the neighbourhood of the equator the pressure contours have a 'reverse S' shape which was found to be very characteristic by Stout and Young (1983). The model results, figure 4 reproduce both features very well. Moreover, they

explain the 'reverse S' shape as resulting from the superposition of the decaying Kelvin wave on the meridional pressure gradients associated with the Hadley circulation.

It is now also clear why no reflected Kelvin waves were found in the models of Anderson (1976) or Bannon (1979, 1982). The antisymmetric heating distributions they chose result in the zonal winds of the Hadley circulation also being antisymmetric about the equator. The forced flow therefore already satisfies the condition (26) of no net flow through the meridian of the boundary, so that no Kelvin wave is generated at the boundary.

6. CONCLUSION

The technique of modelling mountain barriers at the eastern and western boundaries of the oceans by vertical walls appears to give quite realistic simulations of the low level flow. The planetary waves generated at the eastern boundary give a characteristic transition from the meridional boundary flow to the near zonal interior flow. The flow near the western boundary is modified only by the presence of a Kelvin wave, whose amplitude can readily be calculated and so its effect estimated.

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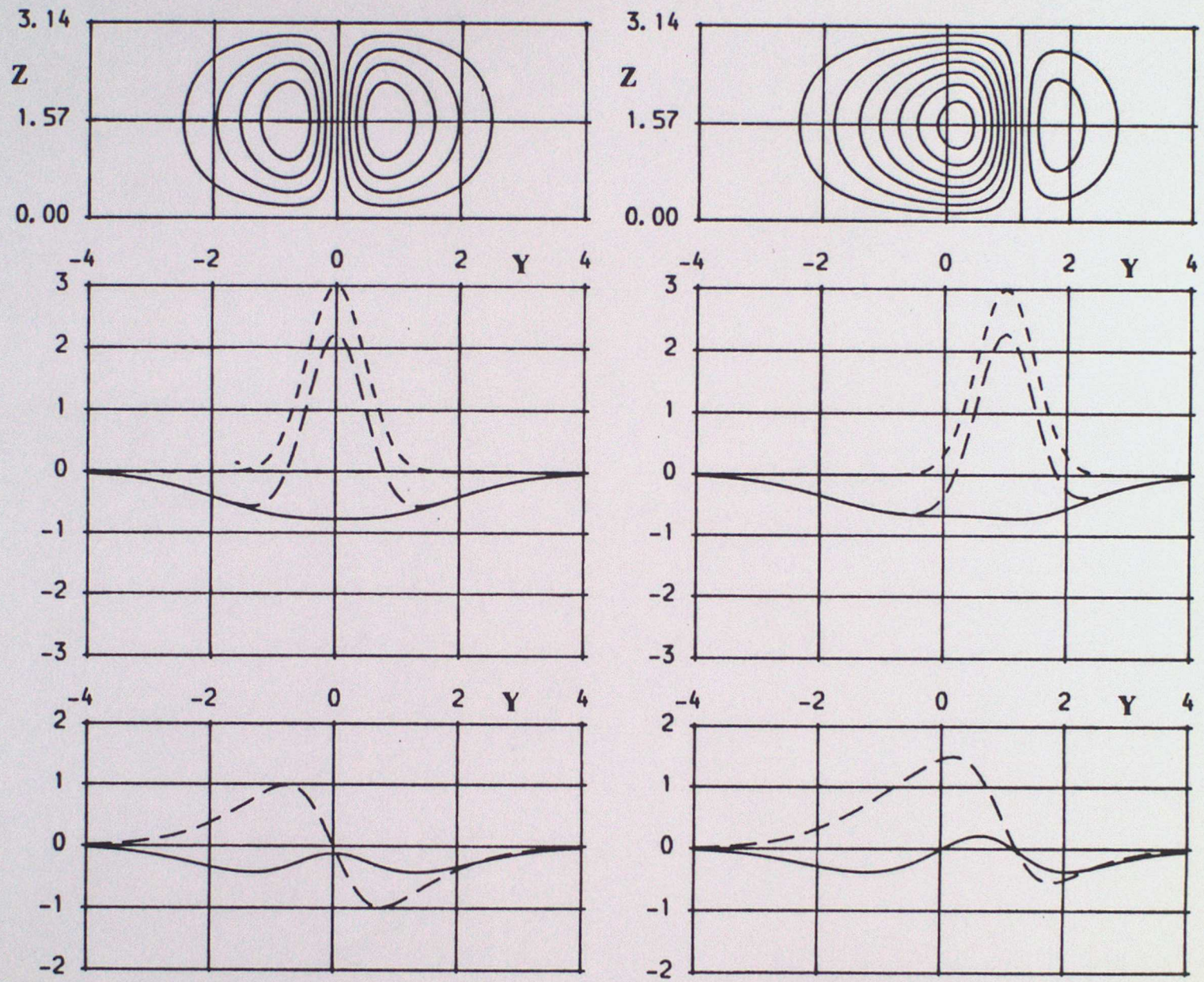
FIGURE CAPTIONS

Figure 1. The Hadley circulation for heating centred on the equator (left), and one Rossby Radius north of the equator (right). The top diagrams show the streamfunction. The variable y is scaled in Rossby radii, and z increases from zero at the surface to π at the tropopause. The middle diagrams show the heating Q (dashed), the vertical velocity w (broken), and the pressure $\epsilon^*\phi$ (full). The bottom diagrams show the zonal velocity $\epsilon^* u$ (full) and the meridional velocity v (broken). (α=3, ϵ^*=0.1, A=0.01).

Figure 2. The low-level flow (wind arrows and pressure contours) in the presence of an eastern boundary at x=0. Heating centred one Rossby radius north of the equator (top), and on the equator (above). The Hadley circulation is as described in section 3 and shown in figure 1. (α=3, ϵ^*=0.1, A=0.01). The variables x and y are scaled in Rossby radii a, a $\sim$ 1000Km.

Figure 3. Components of the flow shown in figure 2 for heating centred one Rossby radius north of the equator, and with an eastern boundary at x=0. Top: the forced Kelvin wave and its reflection. Above: the forced planetary waves and their reflections.

Figure 4. The low level flow in the presence of a western boundary at x=-15. Heating centred one Rossby radius north of the equator (top), and on the equator (above). The Hadley circulation is as described in section 3 and shown in figure 1. (α=3, ϵ^*=0.1, A=0.01).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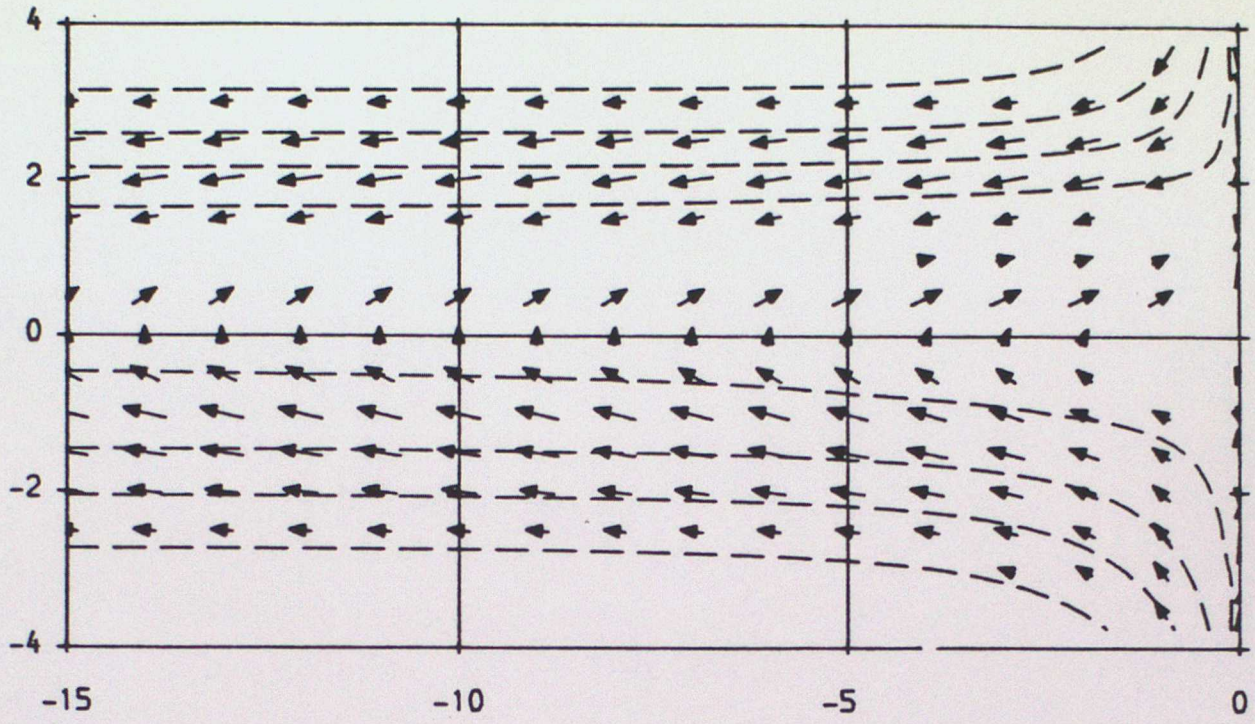
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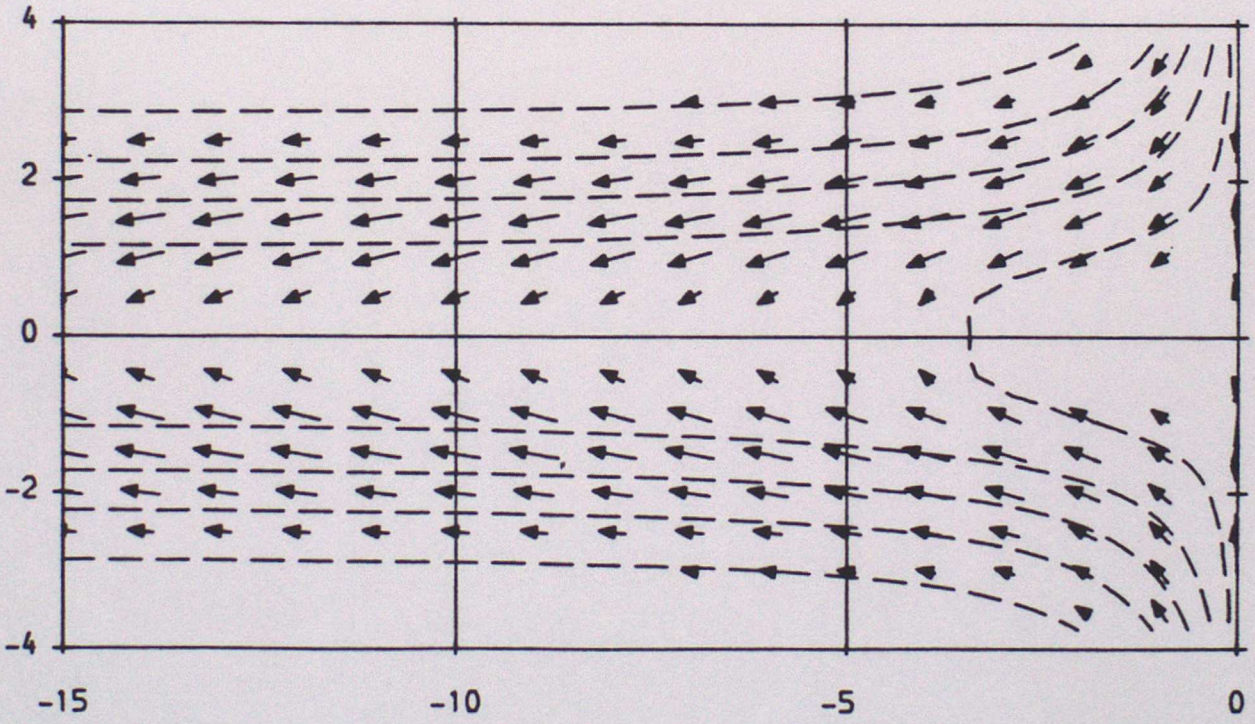
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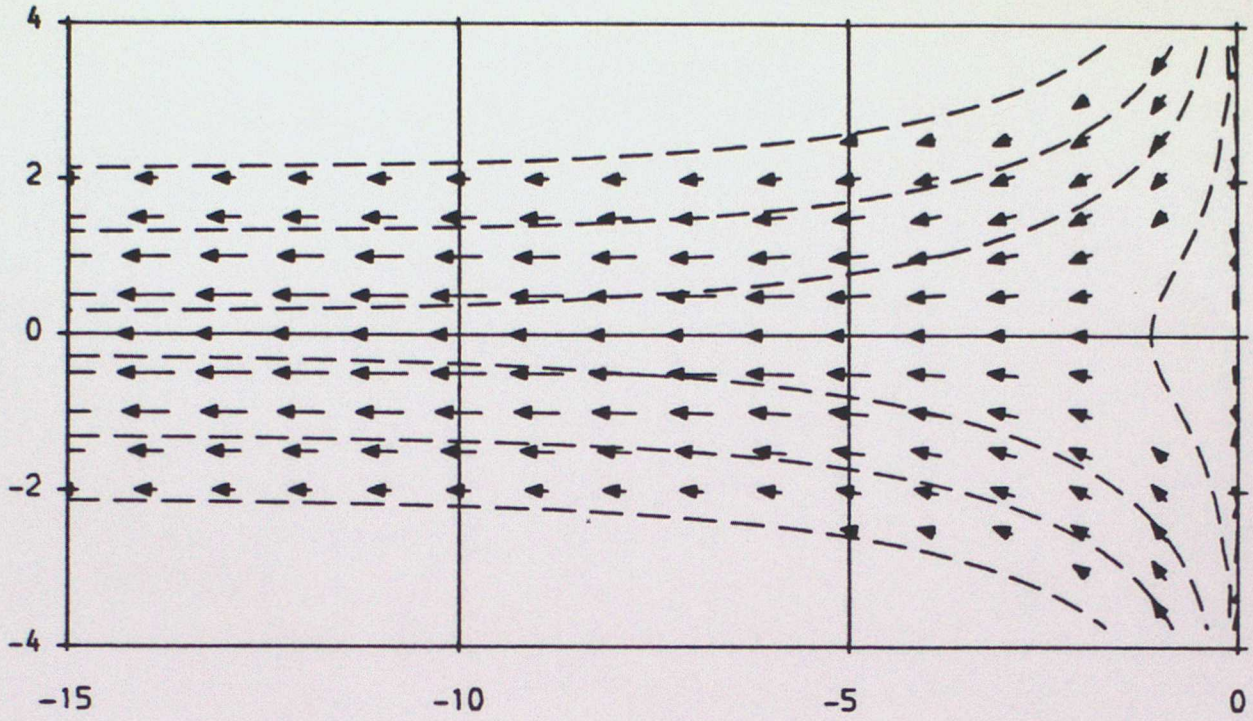
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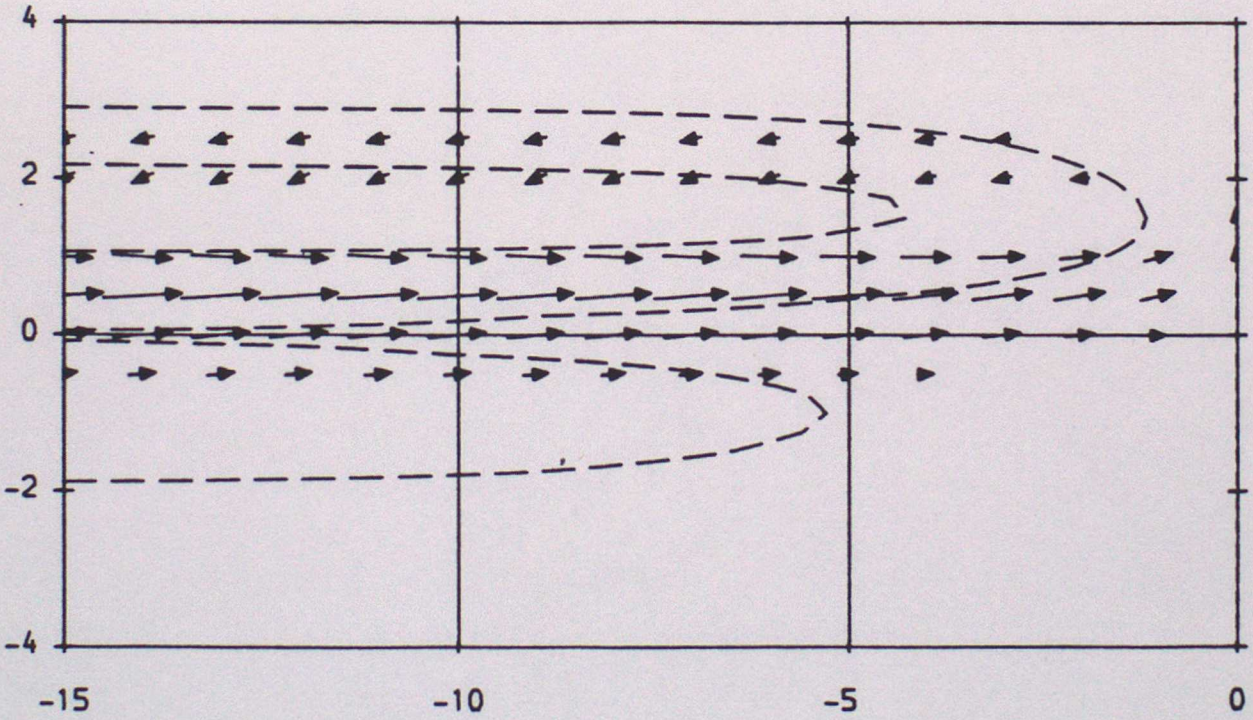
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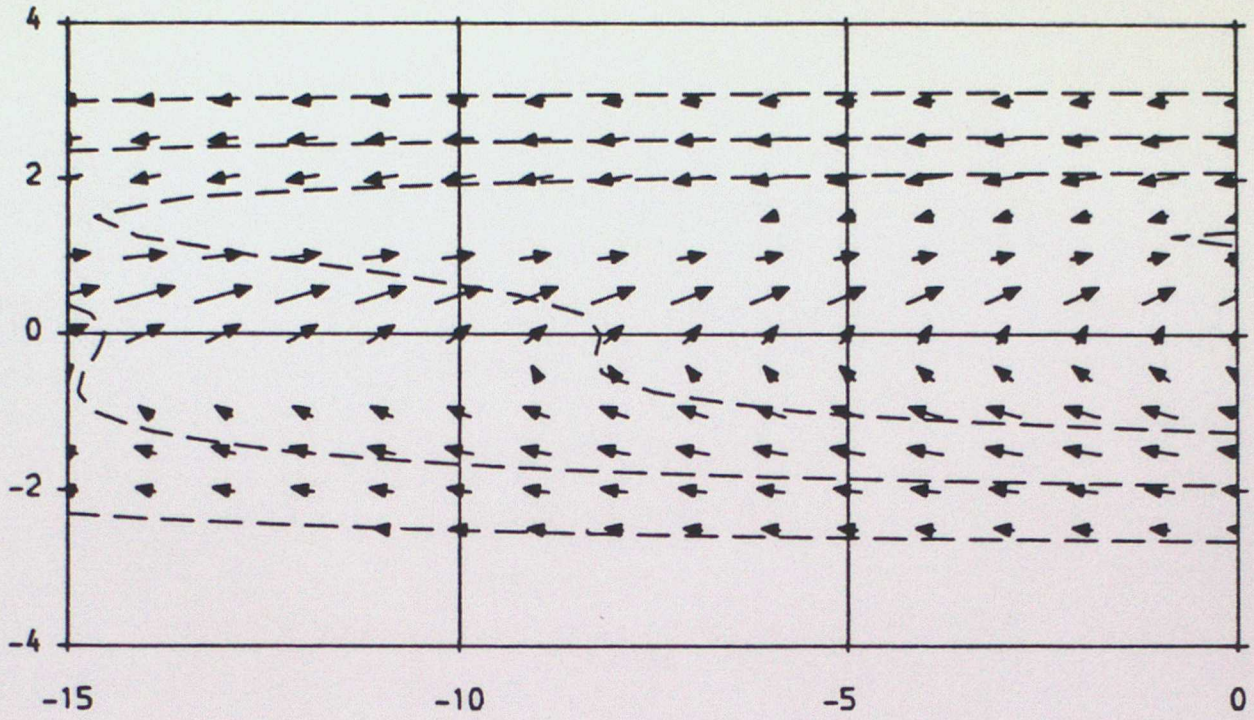
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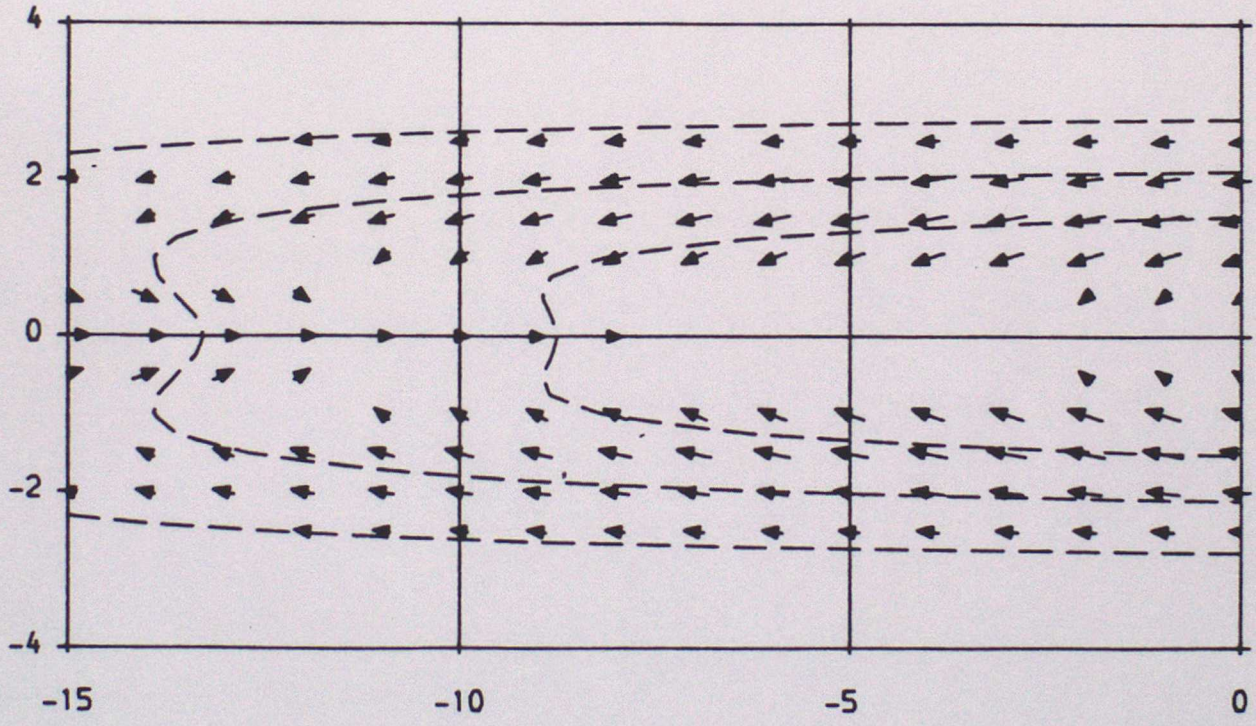
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